Teaching and learning mathematics in a bilingual situation

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Abstract. The aim of this paper is to show how an experiment in teaching maths in both Italian and Friulian in a primary school supports the classical ideas of Vygotskij on the development of language.

Key-words. Elementary maths, primary school, didactical contract, bilingualism.

1. Introduction: The didactical contract. Our present paper is devoted to analyzing the reactions of some primary school pupils involved in a mathematics teaching-learning activity in a bilingual (Italian and Friulian) situation (a more detailed report can be found in Copetti 2005; an earlier experience is described in Agostini et al. 2004).

Our theoretical framework includes the classical researches on the didactical contract, defined as "the set of all behaviours of a teacher which are expected by his/her pupil as well as the set of all behaviours of a pupil which are expected by his/her teacher" (Brousseau 1986). Such a theoretical frame was developed starting with Brousseau's papers (1972, 1981, 1997) on didactical situations and has evolved quite amply (for a general study see: Sarrazy, 1995). For instance the concept of custom (Carbonnier 1971) develops in different terms some peculiarities often associated with the didactical contract (Chevallard 1983, p. 11); the custom can be defined as "a set of mandatory practices [...] of a way of acting established by a habit; most often implicitly" (Balacheff 1988, as quoted in D'Amore 1999).

Our analysis of the experimental situation to which this paper is devoted depends also on the following remark. A more general "pedagogical contract" (Filloux 1973 & 1974) concerns only a teacher and his/her pupil; the didactical contract on the

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other hand involves also the knowledge to be learned (Brousseau 1980, Chevallard 1988, Blanchard-Laville 1989). Our decision to use both Italian and Friulian languages in our teaching-learning activity can be associated with custom. In this paper we examine a few features of the didactical contract connected to each of our two different choices.

It is interesting to recall Vygotskij on the learning process in a minority (mother) language, versus the same process in some other language: development of mother language goes upwards, while development of another language goes downwards. In the first case, basic properties of the language appear at the beginning of the process, and later we have the development of its complex features, linked to awareness of the phonetical structure of the language, its grammatical features and its intentional building. In the latter case, higher properties of the language appear at the beginning of the process, linked to awareness and to intentionality, and later basic properties, referred to free, spontaneous uses of the foreign language (Vygotskij 1990). He also says: these opposite ways are mutually dependent, like the development of scientific and spontaneous concepts (Vygotskij 1990). Our experimental results led us to consider once more his position and, basically, to confirm it.

2. Our didactical experience. Our experimental research was carried out in the Scuola Elementare (primary school) in Venzone (Udine) between December 2004 and April

2005, in a third class (8-9 year old pupils) and in a fourth class (9-10 year old pupils). It consisted of the following activities:

- observing some lessons in mathematics in Italian;
- collecting the features of the didactical contract arising when teaching mathematics in Italian;
- planning and implementing mathematical teaching in Friulian;
- collecting the features of the didactical contract arising when teaching mathematics in Friulian;
- listing advantages and disadvantages of the features of our didactical contract in teaching mathematics in either language;
- a comparison with the features in other subjects.

Our research therefore deals with the problems connected to the didactical contract in mathematics and the role played by bilinguism in this situation. In particular we tried to understand the relation of the didactical contract in mathematics with the didactical contract in the other school subjects in the classes we studied.

We can summarize as follows the questions of interest to us: how do children react when mathematics is taught in Friulian? How and why does the didactical contract change with the language?

2.1 Our experience in a third class. The third class under observation consisted of 14 children: one needs special help of a specialized teacher, who, however did not take part to the maths activities. We considered 11 hours of mathematics in Italian given by the teacher Anita Zamolo (she always spoke Italian, and used Friulian only occasionally to ask her pupils to be quiet) and 4 hours of mathematics in Friulian.

These hours consisted of exercises and problems on the use of the Euro,

on how to compute the double and the halves, and on fractions.

2.1.1 *The features of the didactical contract in Italian*. During the lessons in Italian the following features became apparent:

Features	Subject: maths	Subjects: history, geography,	Subject: (Italian) language	
		natural sciences		
Before starting our activities there are a few task to be				
performed daily	Х	Х	Х	
Our teacher always makes us write down the date	Х	Х	Х	
When I wish to say something I have to raise my hand	Х	Х	Х	
All exercises should be worked out by oneself	Х	Х	Х	
We can use our games as a tool for learning	Х	Х	Х	
Any difficulty should be faced together	Х	Х	Х	
Sometimes our teacher gives us no explanation	Х	Х	Х	
We are requested to be precise when copying from				
the blackboard into our copybook	Х	Х	Х	
All our drawings should be in colour	Х	Х	Х	
We use a pencil to fill in our papers	х	Х	Х	
We can use our own words in writing down definitions	х	Х	Х	
The same word can be used in different frames and				
with different meanings	х	Х	Х	
We are allowed to correct our mistakes	х	Х	Х	
Examples are a tool in solving exercises	х		Х	
Drawings help in visualizing a situation	Х		Х	
An oral discussion precedes the exercises	Х		Х	
Our teacher reads a problem before she dictates it	х			
There is always a procedure to solve any problem	Х			
Any pupil who has difficulty with his problem is asked				
to solve it on the blackboard	Х			
Problems can be structured in different ways	Х			
Data are needed in solving any problem	х			
Practice helps in understanding concepts	х			

2.1.2 The features of the didactical contract in Friulian. The following additional features in the didactical con-

tract arose in our third class during the teaching hours in Friulian.

Features	Subject: maths	Subjects: history, geography, natural science	Subject: (Italian) language es
I am supposed to follow the procedure suggested by			
the teacher	х	х	Х
The correction of our homework is done before			
a new hour starts	Х	Х	Х
The subject remains the same in spite of changing			
the language	х	х	
I am supposed to write in Friulian any exercise			
proposed in Friulian	х	х	
There are several ways to solve a problem	Х		

In particular, the feature "Mathematics remains the same in spite of changing the language" appeared already in our first hour of maths in Friulian. Our children use all strategies learned in the hours given in Italian in solving any problem posed in Friulian. At the end of the hour the children were asked the difference between the "two mathematics" and all made reference to the language only.

The feature "I am supposed to write in Friulian any exercise proposed in Friulian" appeared in all maths hours in Friulian. Pupils write in the language in which a problem has been posed even when the don't speak it at home.

The feature "I am supposed to follow the procedure suggested by the teacher" appeared in our second hour, when in solving an exercise two different procedures could be used. So its emergence was influenced by the chronological organization of our research. Children who spoke Friulian at home followed their teacher's suggestions in solving the exercise. The other children, instead, inserted their teacher's suggestions on how best to solve their exercise in the explanations in Friulian added to clarify their procedure. These latter children paid more attention and spent a longer time in solving their exercises than the others. The emergence of this feature, as well as of the second ("The correction of our homework is done before a new hour starts") might be connected to some uneasiness in some pupils.

2.2 Our experience in the fourth class. Our fourth class consists of 14 children. The teacher Vanda Fadi gave 13 hours of maths in Italian, which were followed by 2 hours in Friulian. The hours in Italian dealt with the different ways in which divisions can be approached (with practical problems) and fractions (with problems and exercises). Usually the teacher spoke in Italian and explained in advance what had to be done.

Features	Subject: maths	Subjects: history, geography, natural science	Subject: (Italian) language es
We are supposed to write down the date before			
any activity	х	х	Х
Our teacher explains what we are going to do	х	х	х
We are supposed to raise our hand before giving our			
opinion	х	х	х
When explaining anything to a fellow pupil I must			
be clear and specific	х	х	х
Sometimes our teacher gives no explanation	х	Х	х
Group recalling activity is helpful for everybody	х	х	х
Our teacher always explains what we are required to do	х	х	х
We are supposed to copy exactly on our copybook			
whatever our teacher explains	х	х	х
When my solution is not the standard one I raise my			
hand and explain my own procedure	х	х	Х
Any difficulty should be faced together	х	х	Х
If anybody is missing we are allowed to occupy his/her place	e x	х	Х
Pupils too are allowed to explain the rules	х		Х
We are not supposed to write down any problem			
the first time our teacher reads it	х		
Everybody works out his problem on his own	х		
We are allowed to ask only our teacher for help	х		
Everybody solves his divisions using his/her own strategies	х		
Pupils describe orally what they are doing when			
working at the blackboard	х		
Problems might include hidden questions	х		
The correction of a problem consists in going to the			
teacher and explaining one's own strategy	х		
Strategy is more important than the result	х		
Any problem must include its data	х		
A table might be helpful in computational mistakes	х		
We can express a fraction in three ways	х		
In fractions a larger denominators means the part is			
smaller	Х		
Any problem can be solved in several ways	х		
Any method I use is valid if it leads to the correct			
solution and if I am able to explain how I implemented	it x		
Explaining my method is helpful to the others	х		

2.2.1 The features of the didactical contract in Italian. The following fea-

tures appeared in the hours given in Italian in our fourth class.

2.2.2 The features of the mathematical contract in Friulian. The follow-Friulian. ing additional features in the didactical contract arose in our fourth class during the teaching hours in

Features	Subject: maths	Subjects: history, geography, natural sciences	Subject: (Italian) language
I try to tackle difficulties by applying the methods I kno	w x	х	Х
I can speak Italian even when the lesson is in Friulian	х		
If a lesson is in Friulian, I write in Friulian on my copybook	х		
Mathematics in Italian and in Friulian is characterized			
by the same procedures	Х		
There are several ways to solve a problem	х		
Friulian is sometimes underrated by children who			
speak Friulian at home	х		

In particular, the feature "I try to tackle difficulties by applying all methods I know" appeared when the pupils received their cards in Friulian. Children that do not know Friulian tried to write their answers by using the words of the supplied texts. A comparison with some features which appeared at the beginning of our investigation (concerning autonomy of methods, and the use of personal strategies) led us to consider a behaviour in evolution.

The feature "I can speak Italian even though the lesson is in Friulian" appeared during the first lesson after the submission of the cards: one child cannot speak Friulian, so he was allowed to speak in Italian in order to get information about the proposed task.

The feature "If a lesson is in Friulian, I write in Friulian on my copybook" appeared when pupils' copybooks were examined: most of the children wrote the date, too, in Friulian; four pupils wrote it in Italian, but only in the first lesson, and unconsciously.

The feature "Mathematics in Italian and in Friulian is characterized by the same procedures" was pointed out during the conversation after the exercises in Friulian. Children compared the "two mathematics" and noticed that only the language is different, and the methods do not change.

The feature "Friulian is sometimes underrated by children who speak Friulian at home" appeared when pupils' answers were analyzed. Children that know Friulian sometimes seemed to underrate some exercises and gave only short answers. Children that understand Friulian but do not speak/write Friulian gave detailed and justified answers. So the above feature can be related to an excessive self-confidence that can cause mistakes.

3. Conclusions. Mathematics remains

the same in spite of changing the language: this conclusion has been reached in both classes examined (pupils' behaviour was rather similar: in fact, the small age difference did not influence our results): in third class, during the first lesson, in Friulian, pupils used the same procedures and strategies as for maths in Italian. In fourth class, children discussed, after exercises in Friulian, differences, if any, and concluded that only the language is different.

Pupils' behaviour depends on didactical contract: children considered exercises in Friulian as a new situation, a game, which can be faced and solved by using tools they already knew well (a mathematical activity in English, a language known by our pupils, could be useful in learning how far their behaviour depends on their understanding the nature of the subject).

Experimental data can support some positions, frequently found in the literature: features of the didactical contract are implicit and shared, they can be changed by an agreement, and they are either general (i.e. common to several subjects) or specific (i.e. specific to a particular subject). In particular, didactical contract in mathematics is specific for every class, shared by both teacher and pupils, referred to the discipline taught and to the language used in exercises.

In particular the behaviour of the pupils when taught mathematics in Italian and in Friulian depends on their knowledge of the language. Mathematics in Italian is considered an everyday situation, so they carry out exercises in conformity with usual rules. Mathematics in Friulian is considered as a new situation: it is an innovation, in spite of the fact that some of them as a rule count in Friulian.

Children whose mother language is Friulian, who speak in Friulian at home and in their environment, underrate mathematical difficulties and focus on the language. So they carried out some exercises incorrectly and gave short or incomplete answers. Children that speak only Italian and understand Friulian are uncertain and tried to give detailed and complete answers; moreover they tried to justify their answers. They wrote in Friulian by using words contained in the text of the problems.

So, with reference to our theoretical framework, Vygotskij's proposition, mentioned earlier, can be confirmed by our experimental data: in the use of mother language, i.e. Friulian, for several pupils, is associated with strongly spontaneous attitude, while awareness and intentionality are mostly referred to the use of Italian.

In particular Vygotskij's quotation can be applied, to both classes, to monolingual children and bilingual children. For bilingual children, the development of Friulian, their mother language, goes "upwards": basic properties of the language appear at the beginning of the process, and later we have the development of its complex features, linked to the conscious use of the language, to its grammatical features and its inten-

tional building. Children learnt Friulian at home, in their environment, and Italian from T.V. and in the nurserv school. They gave just basic answers in early activities: they used only "yes", "no", and numbers. Their behaviour changed after the discussion with monolingual pupils, who gave more detailed answers: in the next exercises they gave complete answers. Some of them made mistakes in the first activity, perhaps underrating difficulties and trying to show a good knowledge of the language. During the discussion, they acknowledged their mistakes and tried to correct them. They work in Friulian, but they count in Italian: as a matter of fact, numbers have been learnt at school in Italian.

For monolingual children, who understand Friulian, the development of the minority language goes "downwards": higher properties of the language appears at the beginning of the process, and later basic properties. They, in activities in Friulian, constructed their answers by using words contained in the text of the problems: so we have verbs incorrectly conjugated, singular words used in plural sense and vice versa, Italianisms and neologisms. Nevertheless, their answers are complete, rich in details and explanations, trying to counterbalance their poor linguistic knowledge. All these pupils carried out exercises correctly. They spent much time in solving problems because of the construction of the answers. They, too, counted in Italian during the discussion.

Bilingual children, who infre-

quently wrote in Friulian, exploited their own oral knowledge of the language in order to write answers to exercises. They strengthened their own writing ability by activities in a minority language. Monolingual children are supported by their knowledge of Italian in writing answers in Friulian. Their sentences reflect structures of Italian corresponding sentences. By that, they enlarge their own Friulian vocabulary and get used to writing in the minority language.

Clearly the pupils took into account, in our experiment, more the linguistic aspects than the mathematical ones. Their behaviour changed when language changed. Pupils consider mathematics in Italian as a traditional subject implying evaluation. Mathematics in Friulian is considered in a different way. Children regarded it as a game activity, not included in the school curriculum and implying no evaluation. Language is not the one and only aspect that causes changes in the didactical contract, although it is certainly an important aspect. The didactical contract changes because the background changes: the awareness of the absence of evaluation implied a decrease of rigour and carefulness in several pupils.

Finally we can express some general considerations: in our opinion, introducing mathematics in Friulian would lead bilingual children to tackle the obstacle of writing (as a matter of fact they use only oral language), and this would need a course of Friulian, taking into account the need to point out some new Friulian terms for mathematical objects. For monolingual children obstacles would be: writing, grammatical rules, comprehension and, moreover, the correspondence of mathematical topics in Italian. In fact, in the middle school, higher school and university mathematics is introduced only in Italian. A mathematical exercise in Friulian would however be useful for all children, since it would enable them to argue in different languages, discover different ways of building sentences, reason in new spheres and develop a dynamic vision of the subject. Further research could point out educational opportunities linked to teaching mathematics in bilingual situations.

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