

Quantum physics in secondary school

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Abstract. A project for the didactic innovation and the teachers' training in quantum mechanics consists of two main phases: traditional main experiments (e.g.: Franck-Hertz, interference, polarisation); approach to Dirac to the theory. We outline a possible strategy to introduce the basic formalism of quantum mechanics without requiring an advanced mathematical or physical background. The "Dirac formulation" of Quantum Mechanics is developed by properly generalizing the description of a simple two-state system, namely the linear polarization of photons interacting with polaroids and birefringent crystals. We also discuss the relation between physical observables and linear operators, a connection which is usually considered as the "hardest" concept in Quantum Mechanics.

A feasibility study has been allowed by an experimentation in the fifth class of a Liceo.

Keywords. Quantum mechanics, didactic innovation, teachers' formation.

1. The basic ideas of our curricular proposal. Quantum Mechanics is usually introduced via a guided tour through historical developments, together with a critical review of some crucial experiments. Besides many advantages (Messiah 1961), this attitude suffers from a serious drawback, especially in elementary treatments where no room is left to go beyond the first rudiment of wave-particle

duality, traditionally followed (or preceded) by a description of the Bohr-like approach to atomic quantization. According to our opinion, it is worthwhile to make a serious attempt to introduce, from the very beginning, what we might call the "Dirac formulation" of Quantum Mechanics (Dirac 1958). The reason is two-fold. First of all, this formulation stresses the role of the superposition princi-

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ple, which is widely recognized as the fundamental principle of Quantum Mechanics, the one requiring the most revolutionary change in our understanding of physical reality; furthermore, the mathematical formalism based on vector spaces and linear operators provides a unifying view of all microscopic phenomena, from the simplest spinning system to the most sophisticated quantized field.

Our proposal is based on the belief that the basic ideas of Quantum Mechanics can be introduced without requiring an advanced mathematical background. The ideal tool to accomplish this task is provided by the interaction of linearly polarized photons with birefringent crystals and polaroids (Baym 1969, Levy-Leblond & Balibar 1990). This phenomenology is so simple that it is almost trivial to discuss as a physical state is conveniently represented by a vector in an abstract vector space. Moreover, the role of linear operators clearly appears as soon as one tries to compute the average value of physical observables related to the polarization of a photon.

The lesson we learn from the physics of polarized photons goes well beyond the description of a simple two-state system. This background permits to introduce some ideas of much wider validity, namely the idea of amplitude, the general meaning of orthogonality between physical states and the quantum description of measuring processes. By taking advantage of these concepts, the transition from the case of polar-

ized photons to an arbitrary physical system is made quite natural.

Actually, we simply sketch the logical path one should follow to make students familiar with what Sakurai (Sakurai 1985) called the “quantum-mechanical” way of thinking.

The following material is reasonably self contained, however we shall often rely on ref. (Ghirardi, Grassi & Michelini 1995 and 1997) from which the reader can grasp the qualitative ideas underlying the superposition principle. Another recent work presents in detail the curricular guideline for the introduction in secondary school of the quantum mechanics formalism (Ragazzon 2000). The same references can be consulted for any detail we skip in the present work.

2. Physical states, amplitudes and vectors. To introduce the idea that quantum-mechanical states are conveniently represented as vectors in an abstract vector space, let us consider the experimental apparatus shown in Figure 1. Basically, it consists of two polaroids with their pass directions along the unit vectors \mathbf{u} and \mathbf{v} . Our attention will be focused on the ensemble Γ of photons filtered by the first polaroid. We know (Ghirardi, Grassi & Michelini 1995 and 1997) that any photon in this ensemble has a well defined physical property of polarisation: that of going undisturbed through a second polaroid with the same pass direction \mathbf{u} . However, in the arrangement of Figure 1, the second polaroid is oriented along the arbitrary direction \mathbf{v} . In such conditions we can only ask for the prob-

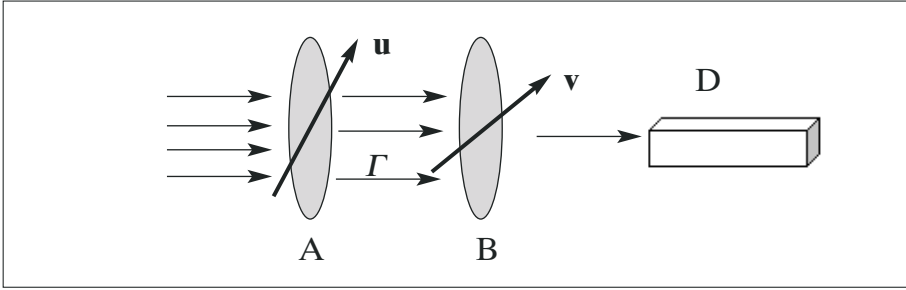


Figure 1. A: polaroid with pass direction along the unit vector \mathbf{u} . B: polaroid with pass direction along \mathbf{v} . D: photon detector. Γ : ensemble of photons filtered by A; the photons of Γ are characterized by a well defined physical property: the property that would enable them to go undisturbed through a second polaroid oriented along \mathbf{u} .

ability $P(\mathbf{u}, \mathbf{v})$ that our photons survive the second polaroid B and trigger the detector D. According to Malus law, the ratio of incident to transmitted light intensity of a beam of linearly polarized light which goes through polaroid B is given by $I_t/I_{in} = \cos^2 \theta$, where θ is the angle between the light polarization and the preferred direction of the polaroid. The ratio I_t/I_{in} can be considered as the ratio of transmitted to incident number of photons. Obviously, this is nothing but the probability we are looking for, thus we can write

$$P(\mathbf{u}, \mathbf{v}) = \cos^2 \theta = (\mathbf{u} \cdot \mathbf{v})^2 \quad (1)$$

For any preferred direction \mathbf{v} , the unit vector \mathbf{u} dictates the statistical behaviour of the photon. If we accept that our predictions are unavoidably of statistical nature, then the unit vector \mathbf{u} provides us with the complete description of the photons contained in Γ . Therefore, the state of a linearly polarized photon is represented by a vector in a two dimensional space.

The vector \mathbf{v} can be used to represent the state of a photon surviving the crossing of polaroid B; thus, if detector D is triggered, we can say that our measure has induced a transition from state \mathbf{u} to state \mathbf{v} . Relation (1) gives us a nice prescription to compute the probability for this transition: it suffices to square the scalar product of the vectors describing the states of the photon before and after the measure.

As for any vector in a two-dimensional space, the state \mathbf{u} can be written as a linear combination of two mutually orthogonal unit vectors, call them \mathbf{H} and \mathbf{V} :

$$\begin{aligned} \mathbf{u} &= \psi_1 \mathbf{H} + \psi_2 \mathbf{V}, \\ \psi_1^2 + \psi_2^2 &= 1 \end{aligned} \quad (2)$$

where the components ψ_1 and ψ_2 are usually referred to as “amplitudes”, obeying the “normalization” condition. Since \mathbf{H} and \mathbf{V} are unit vectors, they also represent two possible states of a linearly polarized photon. Therefore, the vector relation (2) can be re-

garded as the quantitative implementation of the superposition principle for the polarization states of a photon. As far as the amplitudes are concerned, notice that $\psi_1 = \mathbf{H} \cdot \mathbf{u}$ and $\psi_2 = \mathbf{H} \cdot \mathbf{v}$. From equation (1), we then realize that ψ_1^2 and ψ_2^2 give the probability that a photon from ensemble Γ triggers detector D when the second polaroid is oriented along the “horizontal” or “vertical” direction respectively. With a little abuse of language, we can say that ψ_1^2 is the probability of finding a photon in the state \mathbf{H} , while ψ_2^2 gives the probability of finding a photon in the state \mathbf{V} . Obviously, such a statement makes sense only when the second polaroid is properly oriented, otherwise the transition to state \mathbf{H} or \mathbf{V} cannot take place.

Formally, equation (2) has been obtained in a trivial way. However its physical content is far from being obvious. Roughly speaking, one can say that “quantum interference” emerges from the fact that our photons behave as if they were in two distinct states, \mathbf{H} and \mathbf{V} , characterized by mutually exclusive physical properties. Clearly, this is a genuine consequence of the superposition principle, with no classical counterpart. Following J. Bell we could also say that the symbol “+” appearing in equation (2) is conceptually different from “either.... or....”. Further illuminating comments on this point can be found in Ghirardi, Grassi & Michelini 1995 and 1997.

Let us now push our formalism a little further. For later convenience, it is useful to apply decomposition (2) to

vector \mathbf{v} , representing the state of the photons after the second polaroid:

$$\mathbf{v} = \psi_1' \mathbf{H} + \psi_2' \mathbf{V}. \quad (3)$$

The probability $P(\mathbf{u}, \mathbf{v})$ can thus be written as

$$P(\mathbf{u}, \mathbf{v}) = (\mathbf{u} \cdot \mathbf{v})^2 = (\psi_1' \psi_1 + \psi_2' \psi_2)^2. \quad (4)$$

This formally trivial result shows that we can freely switch from a vector description of states to a “representation” of them in terms of amplitudes. In particular, any state of the photon can be represented by a pair of amplitudes; moreover, using equation (4), we can handle amplitudes to compute all transition probabilities relevant to our system. Here, we have deliberately avoided talking about complex amplitudes, since they involve no new physical idea. At any rate, complex amplitudes can be straightforwardly introduced by extending the previous considerations to the case of elliptically polarized photons.

The foregoing results can be generalized to more complex systems in a rather natural fashion. The strategy we suggest is to introduce at this stage the general concept of orthogonality between physical states. First, recall (Ghirardi, Grassi & Michelini 1997) that a state is defined by the properties that can be assigned with certainty to a physical system. Keeping this in mind, the following definition makes sense: two states are said to be orthogonal if they are characterized by mutually exclusive physical properties. The polarization states \mathbf{H} and \mathbf{V} are an example of “physically” or-

thogonal states: if a photon has the property of surviving a polaroid oriented along the horizontal (vertical) direction, then we can surely exclude that it has the property of going through a vertically (horizontally) oriented polaroid. From our definition, it follows that any measuring process can be regarded as a “factory” of orthogonal states. The general formalism of Quantum Mechanics can be obtained by combining together the concept of amplitude and the definition of physically orthogonal states. One expects that the transition probability from one superposition ($\psi_1, \psi_2, \dots, \psi_p, \dots$) to a second superposition ($\psi'_1, \psi'_2, \dots, \psi'_i, \dots$) is obtained from equation (4) by simply letting the index of the amplitudes run from $i=1$ to $i=N$, where N is the number of states we are superimposing.

A diffraction grating is an ideal tool to show that Nature actually forces us to consider the superposition of an arbitrary number of orthogonal states. We consider it relevant to have made plausible the idea that “array of amplitudes”, together with their scalar products, provide us with a useful bookkeeping device to incorporate all the information we need about a physical system.

3. Linear operators and physical observables. In the conventional formalism of quantum mechanics, any physical observable is represented by a linear and hermitean operator. The aim of this section is to make this connection as natural as possible. The first step, of course, is to get some fa-

miliarity with linear operators. In our opinion, it is better to avoid a formal and rigorous treatment of this abstract topic. Rather, one can start by simply writing on the blackboard an object like $\mathbf{a} \mathbf{b} \cdot$ (a sequence of two vectors followed by the symbol of scalar product). At a first sight our new object is rather mysterious. However, one can discover its properties by writing it to the left of an arbitrary vector \mathbf{c} . The result of the operation is $\mathbf{a} \mathbf{b} \cdot \mathbf{c}$. Since $\mathbf{b} \cdot \mathbf{c}$ is just a number, we realize that our guy $\mathbf{a} \mathbf{b} \cdot$ “eats” vectors and produces new vectors proportional to \mathbf{a} .

After some experience with “operators” of the previous type, one can introduce linear combinations of them. In particular, let us define the operator:

$$\hat{O} = \lambda_1 \mathbf{a} \mathbf{a} \cdot + \lambda_2 \mathbf{b} \mathbf{b} \cdot \quad (5)$$

where \mathbf{a} and \mathbf{b} are two orthogonal unit vectors $\mathbf{a} \cdot \mathbf{a} = \mathbf{b} \cdot \mathbf{b} = 1$, $\mathbf{a} \cdot \mathbf{b} = 0$. The action of the operator can be understood by writing its expression (5) to the left of an arbitrary vector \mathbf{c} :

$$\begin{aligned} \hat{O} \mathbf{c} &= (\lambda_1 \mathbf{a} \mathbf{a} \cdot + \lambda_2 \mathbf{b} \mathbf{b} \cdot) \mathbf{c} = \\ &\lambda_1 (\mathbf{a} \cdot \mathbf{c}) \mathbf{a} + \lambda_2 (\mathbf{b} \cdot \mathbf{c}) \end{aligned} \quad (6)$$

The geometrical interpretation of this result is clear. First, the operator projects the input vector \mathbf{c} along the orthogonal directions \mathbf{a} and \mathbf{b} ; such projections are then multiplied by the constants λ_1 and λ_2 ; finally, the new projections are summed to produce the output vector. For later convenience, it is instructive to apply the same operator to vectors \mathbf{a} and \mathbf{b} . As

far as vector \mathbf{a} is concerned, we obtain $\hat{O}\mathbf{a}=\lambda_1\mathbf{a}$. The result is nothing but the same vector \mathbf{a} multiplied by the constant λ_1 : the particular vector \mathbf{a} then obeys the simple transformation law: $\mathbf{a}\rightarrow\lambda_1\mathbf{a}$. Of course, one usually expresses this by saying that \mathbf{a} is an eigenvector of the defined operator with eigenvalue λ_1 . In a similar way we obtain that \mathbf{b} is another eigenvector with the scalar λ_2 as its associated eigenvalue:

$$\hat{O}\mathbf{b}=\lambda_2\mathbf{b}. \quad (7)$$

We have now to establish the connection between linear operators and physical observables. In the context we are dealing with, the only reasonable observables are the polarizations of photons. To say it differently, we can only check whether a photon is polarized along an arbitrary direction \mathbf{v} or not (Ghirardi, Grassi & Michelini 1995 and 1997). A suitable measuring apparatus is sketched in Figure 2; it is

composed by a birefringent crystal which splits a beam of photons in a pair of secondary beams with polarizations along the mutually orthogonal directions \mathbf{v}_1 and \mathbf{v}_2 . The crystal is followed by two photon detectors, one for each secondary beam. The apparatus is then supplemented by an index with two “positions” λ_1 and λ_2 . The index is brought to the position λ_1 (λ_2) if the detector D1 (D2) is triggered. The outcome of our measurement is a random variable, call it λ , which can take only two values, λ_1 and λ_2 .

In dealing with a random variable, we are usually interested in its mean value. The formalism of the previous section enables us to compute the main value of observable λ with no difficulty:

$$\langle \lambda \rangle = \mathbf{u} \cdot \hat{O}_\lambda \mathbf{u}. \quad (8)$$

It should be clear that operator \hat{O}_λ represents a compact and complete

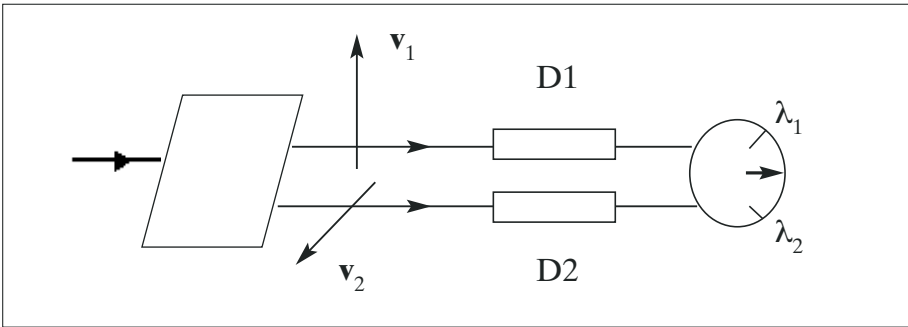


Figure 2. An ideal birefringent crystal splits a beam of photons in a pair of secondary beams with mutually perpendicular polarizations \mathbf{v}_1 and \mathbf{v}_2 . This well known property of birefringent crystals can be used to “measure” the polarization of a photon. If the incident particle is polarized along the \mathbf{v}_1 (\mathbf{v}_2) direction, then detector D1 (D2) is triggered and the pointer is set in position λ_1 (λ_2).

description of the measuring apparatus shown in Figure 2: the possible outcomes of the measure are the eigenvalues of the operator, while its eigenvectors are the possible states in which we can find the photon after the measure.

Once again, it is crucial to realize that our results are not restricted to the simple case of linearly polarized photons (Ragazzon 2000).

The arguments can be repeated for a general physical system, where the states are represented by n -dimensional array of amplitudes.

Our discussion about linear operators and physical observables leaves some fundamental questions without a satisfactory answer. In particular, how can we find the possible values of a physical observable? Furthermore, if such values are unknown, how can we define (and use) the corresponding linear operators? To say it differently: where does the importance of linear operators come from? To answer these questions, we have to keep in mind equation (8) which establishes a simple relation between the average value of a physical observable and its corresponding operator. In addition, we have to recall that the average values of physical observables are assumed to obey the laws of classical Physics, at least in some appropriate limiting situation. This “matching requirement”, together with equation (8), provides a set of conditions which must be fulfilled by the linear operators describing the observables of a physical system. Usually, these constraints are sufficient to specify the action of an operator and to single out its eigenvectors

and eigenvalues. In short, the basic role of linear observables stems from the fact that they represent an efficient tool to “quantize” a system, that is, to accomplish the transition from classical to quantum Physics.

4. A teaching experimentation on the proposed formulation.

A four-year experimentation has been carried out on the most delicate part of the formulation mentioned above for the introduction of the basic ideas of Quantum Physics, connected to the formalism describing it. Such experimentation has always involved the last class of liceo¹ and it has been carried out by studying the interaction of the light with polarizers and birefringent crystals. The experiments with polarizing filters were aimed to recognize that the property of linear polarization according to a given direction is incompatible with the polarization in any other (non orthogonal) direction. The experiments with birefringent crystals have been proposed to stress the meaning of this conclusion, thus supplying a wider phenomenological base and allowing at the same time the discussion around the concepts of quantum indeterminism, the principles of complementarity and indetermination and the non-locality of quantum phenomena. All this has been aimed at helping the students understand the Superposition Principle according to what has been proposed in (Ghirardi, Grassi & Micheli 1995 and 1997). The experiments have been carried out from the teacher’s desk, involving the students with the only available device.

An iconographic representation of the situations and results of the interaction of the light with polarizers and birefringent crystals has been proposed as an element of conceptual synthesis, to be gradually built up by following the results of the macroscopic experiments (out of a large number of photons) Figure 3 (Ghirardi, Grassi & Michelini 1995 and 1997).

The class activity has been carried out with cards showing the schemes of the studied experimental contexts in order to create a synthesis of the results with respect to intensity and polarization of the light transmitted by sequences of two or more parallel polaroids, crossed or placed in different angles, affected by a beam of light or similar situations with birefringent crystals. Every scheme was associated to some close-answers questions or to

some tables where the results of the experiments could be reported, or to some open-answer questions where the students made their hypotheses and the partial conclusions emerged during their class discussions. The cards had a triple aim: quantitative link to the studied phenomenology, summary of the cases related to the interaction between photons with polaroid and birefringent crystals and, last but not least, collection of the conceptual path followed by each student.

During the experimental phase, the students have increasingly elaborated and discussed several hypotheses. As previously foreseen – and in accordance with the objectives of the teaching proposal hereby presented – some concepts emerged, as far as the interpretation of the behaviour of

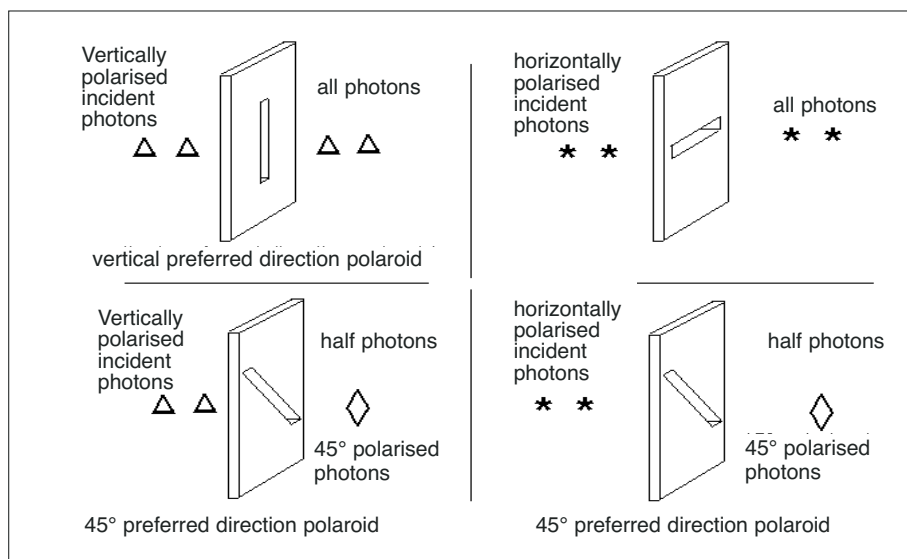


Figure 3. Exemplification of the iconographic representation used.

each single photon with respect to the observed phenomena and, most of all, as far as the photons polarized at 45° . The most common hypotheses about the statistical mixture of states and double property of polarization for a single photon paved the way to the comprehension of the quantic state of the photon and of its changes due to the interaction.

5. Conclusions. After the experimentation discussed so far, some significant conclusions can be drawn as far as this proposal is concerned. The analysed phenomenology is within the capacity of the students and it can be experimented in the classroom by means of commercial instrumentation, which strongly affects the conceptualization phase. The iconographic representation helps the elaboration of the concepts and the exploration of ideas, even though it must not be forgotten that, besides the description of the phenomena, it also implies their interpretation. However, generally speaking, it helps the students as far as the elaboration and expression of their concepts. All

the students realized the possibility of interpreting the intensity of transmission-absorption in terms of relative probability of the single photon. More than the 80% of them has demonstrated to be able to master the passage from the real experiment to the ideal one, as a consequence of a deep mastering of the whole phenomenological experimentation carried out. Mostly (more than the 60%) the students proved to be able to understand the concept of quantum state, the incompatibility of conjugated properties, the non-locality and the quantum indeterminism, even if in different ways and with different awareness. The comparison between classical physics and quantum physics should be further investigated by means of a critical revision (from historical point of view as well) of the new quantum concepts. The experimentation unveiled the need of a wider phenomenological base for the generalization of the approach and the modes allowing the passage to the subsequent formal plan, as already explained in the previous paragraphs.

Notes

¹ The experimentation has been carried out during all the four years at Liceo Scientifico G.

Marinelli of Udine, between May and June. It required seven working-hours in the laboratory plus one hour for the final test.

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